## LINEAR PROGRAMMING (LP) - PRODUCT-MIX EXAMPLE

Consider a product-mix example where one is concerned with what mix of 4 products he should produce during the upcoming week. Production of each product requires a given amount of production time on each of three machines, and each machine has a certain available production time per week. If each product provides certain profit, one need to determine an optimal product-mix so as to maximize profit while considering constraints related to the limited production capacity of machines. Example data for product-mix are given in the following table [Hillier].

|  | Production time per unit (h) |  |  |  | Production time available <br> per week (h) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Product |  |  |  |  |
| Machine for | A | B | C | D | 28 |
| Rolling | 1.7 | 2.1 | 1.4 | 2.4 | 34 |
| Cutting | 1.1 | 2.5 | 1.7 | 2.6 | 21 |
| Welding | 1.6 | 1.3 | 1.6 | 0.8 |  |
| Profit per unit (\$) | 26 | 35 | 25 | 37 |  |

The above described product-mix problem can be formulated as follows:
Maximize $26 x_{1}+35 x_{2}+25 x_{3}+37 x_{4}$
subjectto: $1.7 x_{1}+2.1 x_{2}+1.4 x_{3}+2.4 x_{4} \leq 28$
$1.1 x_{1}+2.5 x_{2}+1.7 x_{3}+2.6 x_{4} \leq 34$
$1.6 x_{1}+1.3 x_{2}+1.6 x_{3}+0.8 x_{4} \leq 21$


The obtained optimization solutions indicate that one should produce 10 units of product B and 5 units of product C so as to obtain maximal profit of $475 \$$. The optimization solution of the LINGO is the same as could be observed from the given report [Hillier].

| Variable | Value | Reduced Cost |
| :---: | :---: | :---: |
| PRODUCE ( P01) | 0.0000000 | 3.577921 |
| PRODUCE ( P02) | 10.00000 | 0.0000000 |
| PRODUCE ( P03) | 5.000000 | 0.0000000 |
| PRODUCE ( P04) | 0.0000000 | 1.441558 |
| Row | Slack or Surplus | Dual Price |
| 1 | 475.0000 | 1.000000 |
| 2 | 0.0000000 | 15.25974 |
| 3 | 0.5000000 | 0.0000000 |
| 4 | 0.0000000 | 2.272727 |

Beside this optimal solution, in BRUTOMIZER® the user is provided with other solutions which are close to optimal solution. For example, production of 7 units of product $B, 6$ units of product $C$ and 2 units of product $D$ yield profit of $469 \$$. Similarly, production of 1 unit of product $A, 8$ units of product B, 5 units of product C and 1 unit of product D yield profit of $468 \$$.

## References:

[Hillier] Frederick S. Hillier, Gerald J. Lieberman, Introduction to Operations Research, McGraw-Hill, 2001.

